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STRESS INTENSITY DRIVEN TOPOLOGY OPTIMIZATION FOR MORPHOGENESIS OF 3D ELASTOPLASTIC STRUCTURES

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ABSTRACT: The purpose of this study is a practical engineering formulation of the topology optimization problem for three dimensional elastoplastic structures. The present study constitutes a comprehensive approach to topology optimization of elastoplastic structures, including both the mechanical problem statement and its efficient computer implementation.
Instead of the traditional approach based on compliance minimization the aim of this work is to find a minimum weight structure, which is able to carry a given load while satisfying the condition that the corresponding stresses do not exceed an allowable limit. The general form of the problem is based on the classical limit design formulations of plasticity. The proposed method finds the optimal structure in an iterative way using only stress intensity distribution and does not require from the User explicit knowledge of any gradients or sensitivities. The effectiveness of the proposed methodology has been illustrated on two representative examples including simply supported and cantilever beams.

Keywords: computational morphogenesis, topology optimization, elastoplastic structures, minimum-weight design, stress constraints.

1. INTRODUCTION

The traditional approach to design of engineering structures relies on a set of candidate structural systems from which, based on the results of finite element analysis, the optimal solution is chosen. The optimal structure found in this way strongly depends on the structural engineer's experience and in the case of newly designed structures of complex functionality, such an approach might not necessarily give a truly optimal solution.

As a result of this, structural topology optimization has been proposed to give an even less experienced designer more flexibility in her or his work. There are two alternative methods for finding structures of optimal topology, namely the continuum based approach and the truss layout method. The former assumes that the whole allowable design space is filled with a solid material from which the optimal topology (a truss-like structure) is extracted. The latter uses a dense grid of nodes connected by bars in all possible ways, from which only the most effective bars are left in the final optimal solution.

Among the civil engineering community topology optimization is also known as 'Structural morphology' or more specifically as 'computational morphogenesis'. As is commonly known, the beginnings of computational morphogenesis are dated at 1904 when the revolutionary work by Michell was published. However, the first topology work was presented by Maxwell in 1870 (Ref.1). Since that time many researchers contributed to this field and it is beyond the scope of this short literature review to mention all of them. For that reason, only the most representative papers are referred to here. However, an excellent comprehensive review of the Michell-type structures can be found in the book by Lewiński et al. (Ref.2).

Essentially, two basic approaches are applied in the case of continuum structures. These are: the fully stressed design (FSD) method (Gallagher, Ref.3, Berke and Khot, Ref.4) and the Solid Material with Penalization (SIMP) method (Bendsoe, Ref.5).

In the first method we tend to obtain such a topology for which stresses within the whole structure are close or equal to the yield limit. The second method assumes that the designed structure is made of a porous material for which effective material properties are represented by density-like parameter based techniques. The FSD is based on the principle, that all elements are fully stressed (in this way the local stress constraints are fulfilled while the SIMP method generally does not use local stress constraints. The modified version of the SIMP–type algorithm is suitable for using local stress constraints.

A method for finding optimal topologies based on the FSD approach has been proposed by Xie and Steven (Ref.6), while Duysinx and Bendsoe (Ref.7) used the SIMP approach.

An additional difference between the FSD and SIMP methods is related to the applied objective function. In FSD, researchers usually minimize the weight of the structure under stress constraints, while in SIMP the objective function is the structure's compliance and constraints are imposed on the volume fraction. An interesting comparison of the minimum weight design under both stress and compliance constraints has been presented by Bruggi and Duysinx (Ref.8).

It is worth noting here that the frequently cited paper by Bendsoe and Kikuchi (Ref.9) showing that the optimal layout can be generated by the representative volume element of a specific porous composite is not fully correct, since the 1st rank composites do not lead to the optimal solution at it was shown by Allaire and Aubry (Ref.10). The first correct papers on compliance minimization of elastic structures were written by Lurie, Cherkaev and co-authors already in the 1970's (Ref.11). Additionally, SIMP utilizes a simplified assumption that the stiffness modulus depends on density through a power law, whereas in other approach such as homogenization this claim is not supported. The homogenized modulus depends on the density in a different manner, rather Pade approximation is necessary, as it was presented in the paper by Tokarzewski and Telega (Ref.12). Besides the above papers assuming linearly elastic material behaviour there are papers investigating elasto-plastic materials (example is paper by Pedersen and Taylor, Ref.13).

Within the International Association for Shell and Spatial Structures (IASS) there is a working group called 'Structural morphology' and its activity since the foundation in 1991 has been described by Motro

(Ref.14). Recent developments in computational morphogenesis can be found in review papers by Ohmori (Ref.15) and Li et al (Ref.16).
The Authors background in the field of topology optimization contains the development of new methods based on optimality criteria (Ref.17) or structural optimization with the aid of graphs (Ref.18). Recently, the Authors extended their approach to topology optimization of structures made of elasto-plastic materials (Ref.19, 20 and 21) and subjected to multiple loading cases (Refs.22 and 23).
At the end of this short overview of different aspects of computational morphogenesis it is worthwhile to mention papers describing the application of topology optimization in architecture and in particular in conceptual design of tall building (Beghini et al Ref.24 and Kazakis et al, Ref.25) or even airplane wings (Aage, Ref.26).
The aim of this work is to present a practical method for topology optimization of 3D continuum structures subjected to stress constraints. Structural weight is considered in this study as the objective function contrary to the classical compliance approach with a volume fraction constraint. The method presented here, although simple, can be successfully used to design optimal structures of various types such as skeletal buildings or stadium roofs. The method is based on determining the stress intensity within the structure and then applying a redundant material removal strategy to the least stressed regions of the structure. This procedure is repeated until the load capacity of the structure may become exceeded.
Finally, the effectiveness of the proposed methodology is demonstrated on several benchmark problems such as a simply supported structure subjected to multiple loading conditions.

2. METHODS, THEORY AND CALCULATION

In the following sections topology optimization under stress constraints will be formulated in continuous domain. Then, the problem will be discretized using standard finite element method and finally some computational aspects will be discussed.

2.1. Topology optimization of elastoplastic structures - problem statement

In the present study we are looking for minimum weight structures made of elastoplastic material. Constraints are imposed on allowable stresses and density.
Mathematically, we can express our approach for topology optimization in the following form

$$\begin{aligned}
&\text{Minimize volume} && V = \int_V \rho(x_i)\, dV \\
&\text{Subjected to} && \int_V \sigma_{ij}\, \delta\varepsilon_{ij}\, dV - \int_\Gamma f_i\, \delta u_i\, d\Gamma = 0 \\
& && d\sigma_{ij} = D^{ep}\, d\varepsilon_{ij} \\
& && \sigma_{HMH} \le \sigma_0 \\
& && \rho_{min} \le \rho \le 1
\end{aligned} \qquad (1)$$

where V is the volume of the structure, σ_{ij} is the stress tensor, $\delta\varepsilon_{ij}$ is the virtual strain, f_i represents external loading, δu_i is the virtual displacement, D^{ep} – elastoplastic material stiffness tensor, σ_{HMH} is the Huber - von Mises - Hencky stress, σ_0 denotes the yield limit and finally $\rho(x_i)$ represents the density of the material distribution.

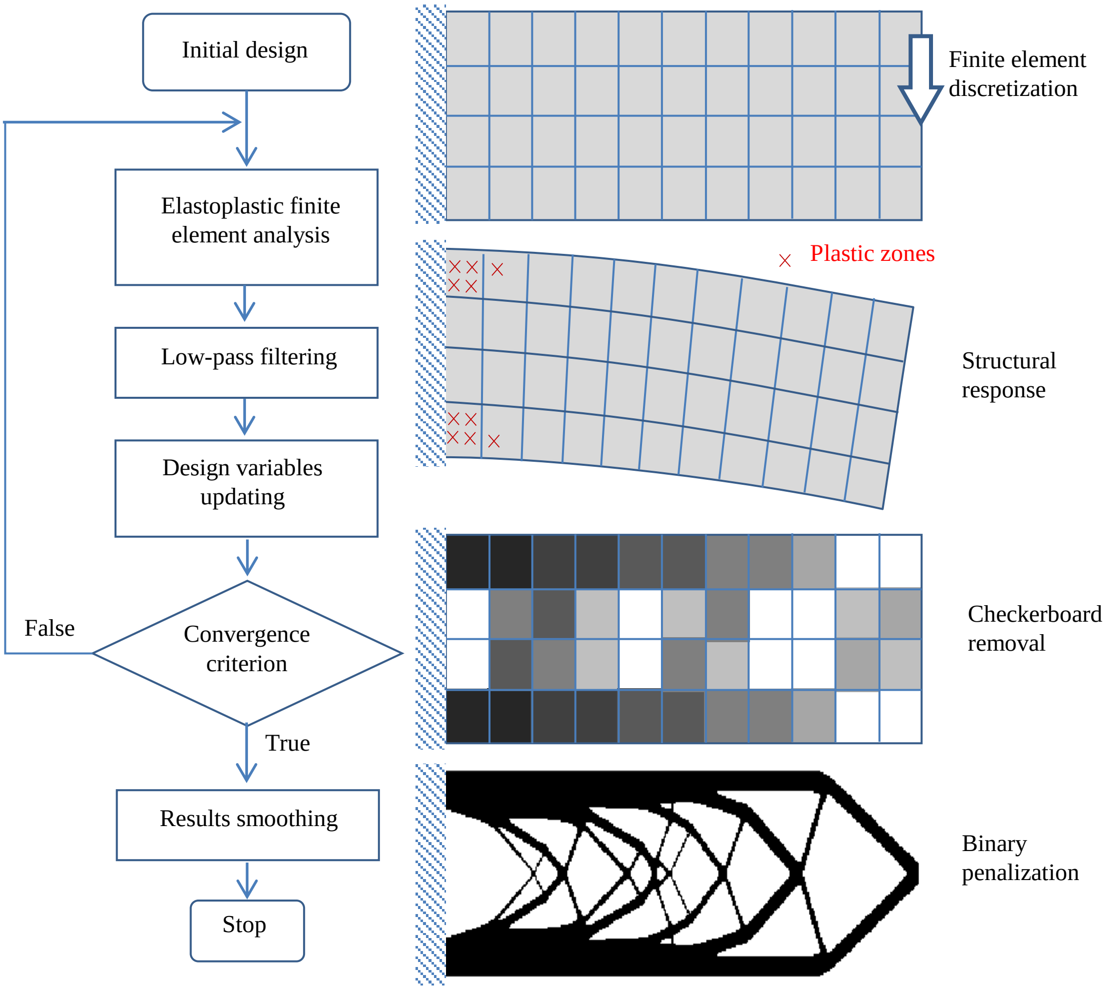


Fig. 1 Computational steps in structural topology optimization

In modern computational mechanics the above continuous formulation of topology optimization is typically replaced with discretized version obtained with aid of the finite element method. Then, instead of (1) the structural topology optimization investigated in this study can be expressed in the following form:

$$\begin{aligned} &\text{Minimize volume} \quad V = \boldsymbol{\rho}^{\mathrm{T}}\boldsymbol{A} \\ &\text{Subjected to} \quad \mathbf{K}(\boldsymbol{\rho})\,\mathbf{u}(\boldsymbol{\rho}) - \mathbf{f} = \mathbf{0} \\ &\qquad\qquad\qquad \boldsymbol{\sigma}/\boldsymbol{\sigma_0} \; \mathbf{-1} \le \mathbf{0} \\ &\qquad\qquad\qquad \boldsymbol{\rho}_{\min} \le \boldsymbol{\rho} \le \mathbf{1} \end{aligned} \tag{2}$$

where $\boldsymbol{A}$ is a vector representing area of individual finite element, $\mathbf{K}(\boldsymbol{\rho})$ denotes tangent stiffness matrix depending on the design variables $\boldsymbol{\rho}$, $\mathbf{u}(\boldsymbol{\rho})$ is displacement vector, $\mathbf{f}$ is external loading vector.
The general flowchart of topology optimization problem is presented in Figure 1. As we can easily observe two main parts in the problem can be identified. The first one is devoted to elastoplastic analysis using finite element method and return mapping algorithm, while the second one is connected to iterative modification of design variables using selected updating scheme.

Algorithm 1. Stress intensity driven topology optimization

Step 1. Initialize *design variables* as a vector of ones $\boldsymbol{\rho}^{(0)} = \{1,1,\ldots,1\}$ and *erased element list* as an empty list $\mathscr{L} = \{\}$.

Step 2. Until load capacity is not exceeded repeat ***Steps 3 to 7***.

Step 3. At *k*-th iteration of the optimization algorithm solve nonlinear equilibrium equations for elastoplastic problem $\mathbf{K}(\boldsymbol{\rho}^{(k)})\,\mathbf{u}(\boldsymbol{\rho}^{(k)}) - \mathbf{f} = \mathbf{0}$ **.**

Step 4. Determine stress intensity vector $\sigma_{\mathrm{avg},e}$ for every finite element $e=1\ldots Ne$ calculated as average of equivalent von Mises stresses $\sigma_{\mathrm{HMH},g}$ evaluated at each Gauss point $g=1\ldots Ng$, then normalize obtained value dividing it by yield limit $\sigma_{\mathrm{avg},e} = 1/(Ng\ \sigma_0) \sum_{g=1\ldots Ng} \sigma_{\mathrm{HMH},g}$.

Step 5. Apply design filter to avoid checkerboard phenomenon. To this end the filtering technique proposed by Sigmund in his famous 99 line code can be used

$$[\sigma_{\mathrm{avg},e}]_{\mathrm{FILT}} = 1/(\rho_e^{(k)} \textstyle\sum_{f=1\ldots Ne} H_f)\sum_{f=1\ldots Ne} H_f\, \rho_e^{(k)}\, \sigma_{\mathrm{avg},f}$$

where the convolution operator H_f is understood as $H_f = r_{\min} - \mathrm{dist}(e, f)$, $\{f$ belongs to the set of elements, for which the following condition is satisfied $\mathrm{dist}(e, f) \le$ given radius$\}$,
The operator dist(e, f) is defined as the distance between the center of element e and center of element f. H_f is zero outside the filter area.

Step 6. Select n finite elements with smallest stresses and assign to their corresponding design variable values to $\rho_{\min}$ i.e. to the lower bound for design variables. Then, add the list of newly selected elements l to the list of previously erased elements $\mathscr{L}^{\mathrm{new}} = \{\mathscr{L}^{\mathrm{new}}; l\}$.

Step 7. Using current list of erased elements $\mathscr{L}$ update corresponding design variables applying the following iterative formula:

$$\boldsymbol{\rho}^{(k+1)} = \boldsymbol{\rho}^{(k)}[\boldsymbol{\sigma}_{\mathrm{avg}}]^p$$

where p is penalty factor to avoid porous intermediate solution. Three different cases are distinguished here:

a) if $\boldsymbol{\rho}^{(k)}[\boldsymbol{\sigma}_{\mathrm{avg}}]^p \le \boldsymbol{\rho}_{\min}$
then $\boldsymbol{\rho}^{(k+1)} = \boldsymbol{\rho}_{\min}$

b) if $\boldsymbol{\rho}^{(k)}[\boldsymbol{\sigma}_{\mathrm{avg}}]^p \ge \mathbf{1}$
then $\boldsymbol{\rho}^{(k+1)} = \mathbf{1}$

c) if $\boldsymbol{\rho}^{(k)}[\boldsymbol{\sigma}_{\mathrm{avg}}]^p \ge \boldsymbol{\rho}_{\min}$ and $\boldsymbol{\rho}^{(k)}[\boldsymbol{\sigma}_{\mathrm{avg}}]^p \le \mathbf{1}$
then $\boldsymbol{\rho}^{(k+1)} = \boldsymbol{\rho}^{(k)}[\boldsymbol{\sigma}_{\mathrm{avg}}]^p$

3. RESULTS

To check correctness of the implemented algorithm several benchmark examples have been calculated. The first example is a plane stress structure with dimension 30m x 10m (fig. 2) and loaded in the middle at top edge. Structure are simply supported with the booth degree of freedoms blocked at the left bottom corner and the roller at the right bottom corner.

Material parameters in example 1 are following:
E = 34.474 GPa
ν = 0.11
ρ = 568.7 kg/m^3
σ_0 = 210 MPa
thickness = 0.2286 m
penalty parameter = 1.75
filter radius = 2

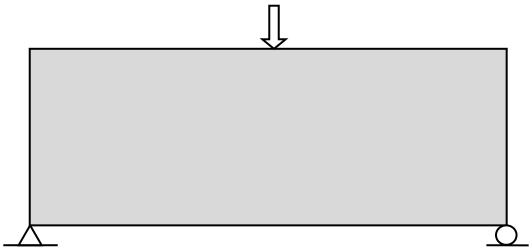


Fig.2 Design domain and boundary conditions for example 1.

Two load versions of load has been applied. First one (Fig.3) is 0.5 x collapse load and second one 0.95 x collapse load (Fig.4). Both solutions ale similar with literature results although second one with load closer to collapse load is significantly thicker.

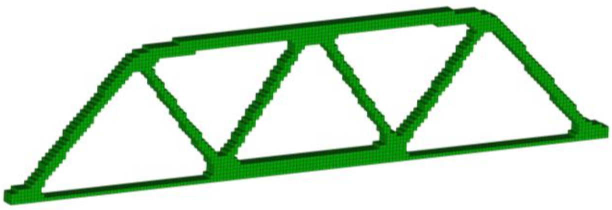


Fig.3 Optimal topology for load equal to 0.5 x collapse load.

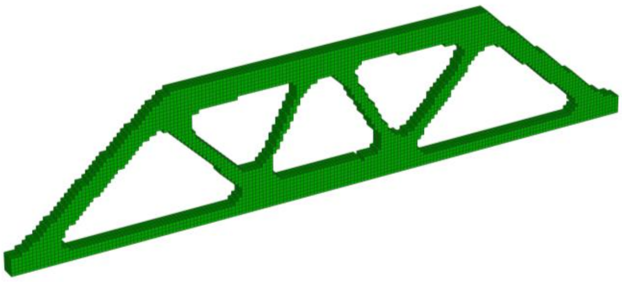


Fig.4 Optimal topology for load equal to 0.95 x collapse load.

Next example is a cantilever with force acting at its end (Fig.5). Dimensions of design space are 20m x 10m. This example is also computed using the above methodology.

Fig.5 Design domain and boundary conditions for example 2.

Material parameters used in this second example are as follows:
E = 34.474 GPa
ν = 0.11
ρ = 568.7 kg/m^3
σ_0 = 210 MPa
thickness = 0.2286 m
penalty parameter = 1.75
filter radius = 2.

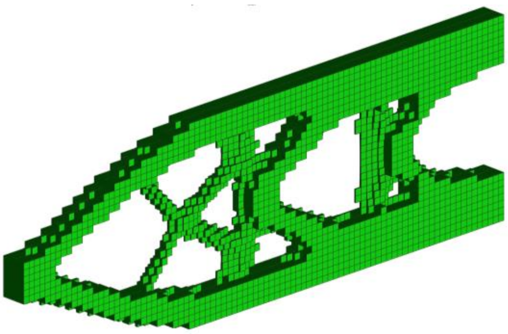


Fig.6 Optimal topology of a cantilever structure for load factor 0.5.

This example also has been computed with two load version one with half value of collapse force (fig.5). We observe good agreement with similar solution taken from the literature. In the case of a plastic-elastic material, the load value significantly influences the results. The more the load approaches the value of the collapse force, the thicker the topology arises. On figures 6 and 7 we present optimal topologies for load factors equal to 50 and 95 per cent of the ultimate load of initial topology, respectively.

The third example illustrates the parametric studies of the variation of the size of the penalty parameter and the filter radius smoothing the chessboard effect. The task data are the same as in the first example.

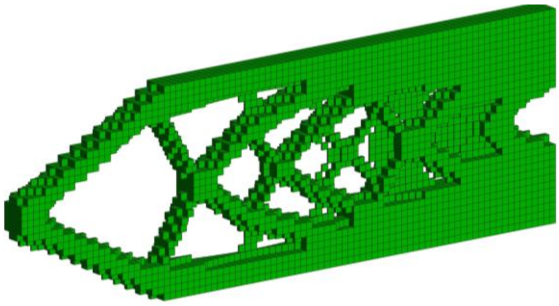
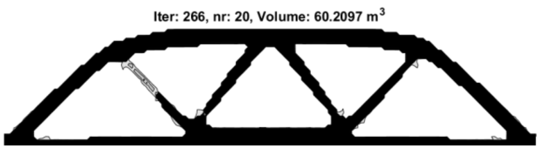
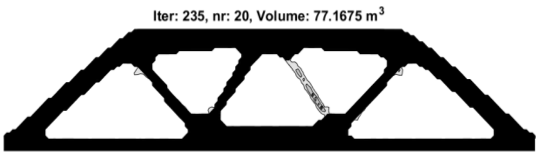
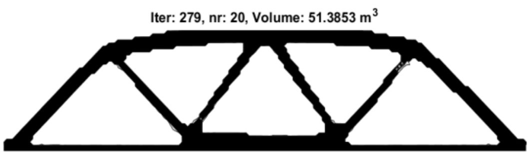
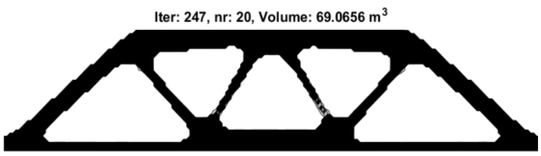
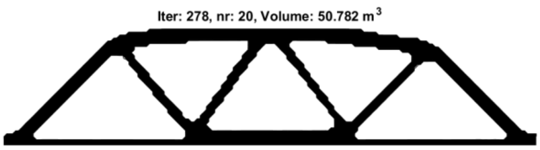
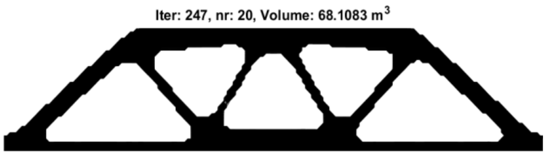
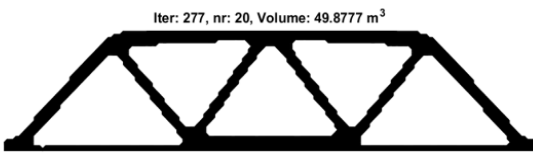
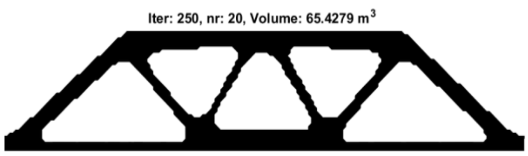


Fig.7 Optimal topology of a cantilever structure for load factor 0.95.

Material parameters of the third example are:
E = 34.474 GPa
ν = 0.11
ρ = 568.7 kg/m^3
σ_0 = 210 MPa
thickness = 0.2286 m

The variation of the smoothing radius works more similarly as the load change. The larger the radius of the smoothing filter, the thicker the topology we get. The penalty parameter also affects the thickness of the structure but less than the radius of the smoothing filter. The smaller the penalty parameter, the thinner the design (Table 1).

Table 1. Influence of the filter radius and penalty factor on optimal volume

Penalty	Filter radius	
	4	6
1.5		
1.75		
2.0		
3.0		

4. CONCLUSIONS

In the paper a practical approach to structural topology optimization known also as a '*computational morphogensis*' has been proposed. The overall method is able to deal with 2D and 3D elastoplastic problems subjected to stress constraints.

The presented contribution consists of a novel formulation of structural topology optimization problem, computer implementation using a high-level programming language and numerical examples. All the above contributions proved that the method can be successfully applied in solving realistic engineering problems and replace the classical compliance minimization technique.

The computer implementation proposed in this study demonstrated that our stress intensity driven topology optimization can be utilized in two ways:

a) it can be easily adapted to existing FE codes, expanding their functionality, or

b) it can be implemented from scratch in languages for technical computing such as MATLAB.

Following the second route, a prototype program has been developed to demonstrate the effectiveness of the proposed method for computational morphogenesis.

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